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# THE APPROXIMATE TOLERANCE LIMITS FOR $\hat{C}_p$ CAPABILITY CHART BASED ON RANGE USING α -CUTS TRAPEZOIDAL FUZZY NUMBERS

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## ABSTRACT

Process Capability Indices (PCI) is widely used to determine whether the production process can produce product within the specified limits. Different methods have been carried out in order to estimate process capability in the literature. Most of the estimated capability indices are based on the assumptions of simple sample of observation from normally distributed process, which is in control and may give incorrect conclusions when the estimator using simple sample. So, here several small subsamples to make decision regarding the process capability and distribution of estimated with sub grouping are taken for consideration. Patnaik's approximation of chi-square sampling distribution of the $\hat{C}_p$ is used to assess the process performance. To improve the process performance, the fuzzy trapezoidal number estimation of tolerance limits for $\hat{C}_p$ capability chart based on range using α –cuts is constructed. An application is also presented and the flexibility of control limits is increased.

**KEYWORDS:** Range Tolerance Limits, Range α –Cuts Charts, Trapezoidal Fuzzy Process Capability Chart, Trapezoidal Fuzzy Range Chart

## INTRODUCTION

Process capability indices are widely used to check whether the production process performance is within the customer's requirement. A process meeting customer requirement is called "capable". A process capability (PCI) is a process characteristic relative to specifications. The setting and communication becomes vey simpler and easier by using process capability indices to express process capability between manufacturers ad customers. The use of these indices provides a unit less language for evaluating not only the actual performance of production processes, but the potential performance as well. The indices are intended to provide a concise summary of importance that is readily usable. Engineers, manufacturers and suppliers can communicate with this unit less language in an effective manner to maintain high process capabilities and enable cost savings. The capability of a process and effectiveness of control charts are directly related. For simplifying practical usage of $\hat{C}_p$, it is proposed a new method of construction of fuzzy trapezoidal number estimation of tolerance limits for $\hat{C}_p$ capability chart based on range using α –cuts. It is very easy to construction as well as for making interpretations like a Shewhart traditional charts.

In this paper, the study is structured in the following order. Firstly, the literature reviews of various Process capability Indices were discussed. Secondly, the approximate tolerance limits for $\hat{C}_p$ based on range are constructed by using chi-square distributions. Thirdly, the approximate tolerance limits for $\hat{C}_p$ based on range is transformed to fuzzy trapezoidal number tolerance limits for $\hat{C}_p$ based on range by using α-level fuzzy midrange transform technique. Fourthly, the fuzzy estimations of $\hat{C}_p$ chart based on range by using α-cuts are constructed. Next, the applications to understanding the fuzzy estimations of $\hat{C}_p$ chart based on range by using α-cuts are given and finally, the conclusions are presented.

## LITERATURE REVIEWS

There are different indices that are given in literature review. In this case the quality characteristics X and the corresponding random sample ($x_1$, $x_2$,$x_3$,……$x_n$) are normal, in fact $X \sim N(\mu, \sigma^2)$. Let LSL and USL denotes the lower and upper specification limits, $M = \frac{(\text{USL} + \text{LSL})}{2}$ , the midpoint of tolerance interval (LSL, USL), t the target value for μ, which we assume that t = m.

Kane (1986) was suggested the simplest and process potential index defined as

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

This index is a ratio of tolerance region to process region. It is clear that this method consider the variation of process. 6 $\sigma$ in denominator of above fraction are based on assumption of approximately normal of data. It will react to change in process dispersion but not change of process location.

Sullivan (1984) has been suggested a new process index $C_{PK}$ in order to reflect departure from the target value as well as change in the process variation, is given by

$$C_{pk} = \frac{Min\ (USL - \mu,\ \ \mu - LSL)}{3\sigma} \tag{2}$$

Chan, Chen and Springe (1988) given another new process index $C_{PM}$ , in order to be a sign of departure from the target value as well as change in the process variation, defined and is given by

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - t)^2}} \tag{3}$$

6σ, 3σ in denominator of above fractions are based on assumption of approximately normal of data. In the other words main constraints in above indices is its normality assumptions. Departure from the target value carry more weight with the other well known capability indices is defined by

$$C_{pmk} = C_p(1 - k) \text{ where } k = \frac{|\mu - M|}{(USL - LSL)/2} \tag{4}$$

$C_{pm}$ and $C_{pmk}$ react more both in dispersion and location than $C_{pk}$. $C_{pmk}$ is more sensitive than $\mathbf{C_{pm}}$ to deviations from the target value T.

Kerstin Vannman (1995), has been given the unified approach. Vannman constructed a superstructure class to include the four basic indices $C_p$, $C_{pk}$, $C_{pm}$ and $\mathbf{C_{pmk}}$ as special cases. By varying the parameters of this class, we can find indices with different desirable properties. The proposed new, indices depend on two non-negative parameters, u and v, as

$$C_p(u, v) = \frac{d - u|\mu - M|}{3\sqrt{\sigma^2 + v(\mu - T)^2}} \tag{5}$$

It is easy to verify that:

$$(0,0) = C_{p;} \qquad C_p(1,0) = C_{pk} \quad C_p(0,1) = C_{pm;} \qquad C_p(1,1) = C_{pmk;}$$

Form the study of $C_p(u,v)$, large value of u and v will make the index $C_p(u,v)$ more sensitive to departure from the target value. A slight modification gives general index class which includes

$$C_p(u_1,u_2,v) = \frac{d-u_1|\mu-M|-u_2|T-M|}{3\sqrt{\sigma^2+v(\mu-T)^2}} \tag{6}$$

$$C_p(0,1,1) = C^*_{pm}$$

The five $C_p, \ C_{pk}, \ C_{pm}, \ C_{pmk} \ and \ C^*_{pm}$ are equal when $\mu = T = M$, but differ when $\mu \neq T$.

Carr (1991), The other approach is defined to use non conforming ratios as an index for capability process for the first time

$$NC = p = p\{x \notin [L,U]\} = 1 - \{\varphi\left(\frac{U-\mu}{\sigma}\right) - \varphi\left(\frac{L-\mu}{\sigma}\right)\} \tag{7}$$

This approach based on the stepwise loss function as below:

$$loss = \begin{cases} 0 & L \leq X \leq U \\ 1 & otherwisw \end{cases}$$

Fleig (2000), suggested and defined by Using fraction conforming as 1-NC (nonconforming). Clements (1989), in an influential paper, suggested that “6σ” be replace by the length of the interval between the upper and lower 0.135 percentage points of the distribution of X and defined

$$C'_p = \frac{U-L}{(\varepsilon_{1-a}-\varepsilon_a)} \tag{8}$$

Greenwich and Jahr-Schaffrath (1995) defined the index $C_{pp}$ which provides an uncontaminated separation between information concerning process accuracy and process precision as follows

$$C_{pp} = C_{ia} + C_{ip} \tag{9}$$

Where the inaccuracy index $C_{ia} = (\frac{\mu-T}{\sigma})^2$ and

Imprecision index $C_{ip} = (\frac{\sigma}{D})^2$ and $\mathrm{D} = \min(\mathrm{T}-\mathrm{LSL}, \mathrm{USL}-\mathrm{T})/3.$

Bernardo and Irony (1996), The bayes capability index $C_{B(D)}$ given by

$$C_{B(D)} = \frac{1}{v}\Phi^{-1}\{\Pr(y \in A/D)\} \tag{10}$$

A Bayesian index is proposed to evaluate process capability which within a decision –theoretical framework, directly assesses the proportion of future work may be expected to lie outside the tolerance limits.

Hsin-Lin Kuo (2010) extended the capability indices by getting approximate tolerance limits for **C*p*** capability chart based on Range

**Approximate Tolerance Limits for $\hat{C}_p$ Based on Range Using Chi-Square Distribution**

Let $X_1, X_2, X_3, \ldots\ldots, X_n$ be a random sample of size n drawn from a normal population with mean μ and standard deviation σ. The range of this sample is defined by $R = X_{max} - X_{min}$

Suppose the total samples are grouped into m subsamples such that each subsample contains n observations. The mean of the m ranges will be denoted by $\bar{R}_{m,n}$ and the range of a single sample of size n is denoted by $R_{1,n}$.

$\bar{R}_{m,n}/d_2$ is the unbiased estimator of σ, where $d_2$ and $d_3$ are constants. According to Patnaik, it has been shown that $\bar{R}_{m,n}/\sigma$ is approximately distributed as $\frac{c\sqrt{\chi^2}}{\sqrt{v}}$. That is

$$(\bar{R}_{m,n}/\sigma)^2 \equiv C^2\frac{\chi_v^2}{v} \quad \text{and} \quad (\bar{R}_{m,n}/\sigma)^2 \ \mathrm{X}\ \frac{v}{c^2} \equiv \chi_v^2$$

Where $\chi_v^2$ denotes a chi-square distribution with $v$ degrees of freedom, and c and $v$ are constants which are functions of the first two moments of the range variable, given by

$$v = \frac{1}{(-2 + 2\sqrt{1 + 2(d_3/d_2)^2/m}}$$

$$c = d_2 \ \mathrm{x}\ \sqrt{\mathrm{v}/2}\ \mathrm{x}\,\Gamma(\mathrm{v}/2)\,/\,\Gamma((v+1)/2) \approx d_2\,(1 + 1/(4v))$$

Using this relations, the values of c and $v$ can be easily obtained for any n and m. Assume that the process the measurement follows N (μ, $\sigma^2$), the normal distribution, the $\hat{C}_p$. index are given below

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

Apply a simple approximation procedure based on range we can obtain the tolerance limits of the estimator of $\hat{C}_p$. The 100(1 – α) approximate tolerance limits for $\hat{C}_p$ together with R charts

$$1 - \alpha = \mathrm{P}\left(\chi^2_{1-\frac{\alpha}{2}} \le \chi^2_{\mathrm{R}} \le \chi^2_{\frac{\alpha}{2}}\right) = \mathrm{P}\left(\mathrm{J}_1\bar{\mathrm{C}}_\mathrm{p} \le \bar{\mathrm{C}}_\mathrm{p} \le \mathrm{J}_2\bar{\mathrm{C}}_\mathrm{p}\right)$$

$$\text{Where} \quad \mathrm{J}_1 = \frac{\mathrm{d}_2}{\mathrm{c}}\ \mathrm{x}\ \sqrt{\frac{\mathrm{v}}{\chi^2_{\alpha/2}}} \quad \text{and} \quad \mathrm{J}_2 = \frac{\mathrm{d}_2}{\mathrm{c}}\ \mathrm{x}\ \sqrt{\frac{\mathrm{v}}{\chi^2_{1-\alpha/2}}}$$

And $\chi^2_{\alpha/2}(v)$ is the upper $\alpha/2$ quantile of the chi-square distribution with *v* degrees of freedom. So, the 100(1 – α) approximate tolerance limits for $\hat{C}_p$ based on range is given by $(J_1\hat{C}_p, J_2\hat{C}_p)$

where $\hat{C}_p = \frac{USL-LSL}{6(\bar{R}/d_2)}$

Approximate upper tolerance limit: $J_2\hat{C}_p$

Center line: $\hat{C}_p$ (11)

Approximate lower tolerance limit: $J_1\hat{C}_p$

**Fuzzy Transformation Techniques and α - Level Fuzzy Mid – Range**

There are four Fuzzy transformation techniques, which are similar to the descriptive measure of central tendency: α- level fuzzy mid – range, fuzzy median, fuzzy average and fuzzy mode. In this study, α- level fuzzy mid – range transformation techniques used for the construction of approximate tolerance limits for $\hat{C}_p$ based on range to fuzzy trapezoidal number control chart. The α- level fuzzy mid – range $f_{mr}^{\alpha}$ is defined as the midpoint of the ends of the α- cuts. An α- level cut, denoted by $A^{\alpha}$ , is a nonfuzzy set that comprises ale element whose membership is greater than or equal to α. If $a^{\alpha}$ and $b^{\alpha}$ are the end points of $A^{\alpha}$ , then

$$f_{mr}^{\alpha} = \frac{a^{\alpha} + b^{\alpha}}{2} \quad (12)$$

α- level fuzzy mid – range of the sample is given by,

$$S_{mRj}^{\alpha} = \frac{\hat{C}_{paj} + \hat{C}_{pdj} \ \ +\alpha[(\hat{C}_{pbj} - \hat{C}_{paj}) - (\hat{C}_{pdj} - \hat{C}_{pcj})]}{2} \quad (13)$$

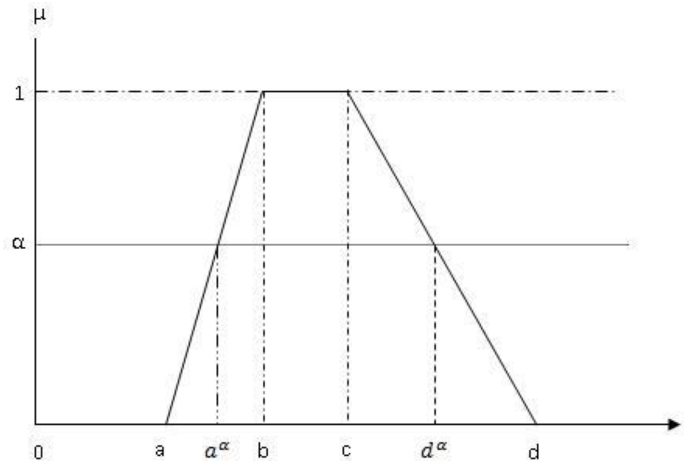


**Figure 1: Representation of a Sample by Trapezoidal Fuzzy Numbers**

**Fuzzy approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

Fuzzy approximate tolerance limits for $\hat{C}_p$based on Range can be easily in a similar way using trapezoidal fuzzy numbers are as follows.

Approximate upper tolerance limit: $\{J_2\hat{C}_{pa}\ , J_2\hat{C}_{pb}\ , J_2\hat{C}_{pc}, J_2\hat{C}_{pd}\ \}$

Center line: (14)

$$\left\{\hat{C}_{pa} = \frac{USL-LSL}{6\left({}^{R_a}/_{d_2}\right)}, \hat{C}_{pb} = \frac{USL-LSL}{6\left({}^{R_b}/_{d_2}\right)}, \hat{C}_{pc} = \frac{USL-LSL}{6\left({}^{R_c}/_{d_2}\right)}, \hat{C}_{pd} = \frac{USL-LSL}{6\left({}^{R_d}/_{d_2}\right)}\right\}$$

Approximate lower tolerance limit: $\{J_1\hat{C}_{pa}\ , J_1\hat{C}_{pb}\ , J_1\hat{C}_{pc}\ , J_1\hat{C}_{pd}\ \}$

**α –Cut Fuzzy Approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

α **–cut f**uzzy approximate tolerance limits for $\hat{C}_p$ based on Range can be calculated as follows.

Approximate upper tolerance limit**:** $\{J_2\hat{C}^{\alpha}_{pa}\ , J_2\hat{C}_{pb}\ , J_2\hat{C}^{\alpha}_{pc}, J_2\hat{C}^{\alpha}_{pd}\ \}$

Center line: (15)

$$\left\{\hat{C}^{\alpha}_{pa} = \frac{USL-LSL}{6\left(\bar{R}^{\alpha}_{a}/d_2\right)}, \hat{C}_{pb} = \frac{USL-LSL}{6\left(\bar{R}_{b}/d_2\right)}, \hat{C}_{pc} = \frac{USL-LSL}{6\left(\bar{R}_{c}/d_2\right)}, \hat{C}^{\alpha}_{pd} = \frac{USL-LSL}{6\left(\bar{R}^{\alpha}_{d}/d_2\right)}\right\}$$

Approximate lower tolerance limit: $\{J_1\hat{C}^{\alpha}_{pa}\ , J_1\hat{C}_{pb}\ , J_1\hat{C}^{\alpha}_{pc}, J_1\hat{C}^{\alpha}_{pd}\ \}$

Where

$$\hat{C}^{\alpha}_{pa} = \hat{C}_{pa} + \alpha(\hat{C}_{pb} - \hat{C}_{pa}) \tag{16}$$

$$\hat{C}^{\alpha}_{pd} = \hat{C}_{pd} - \alpha(\hat{C}_{pd} - \hat{C}_{pc}) \tag{17}$$

**α- Level Fuzzy Midrange Approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

α- level Fuzzy midrange approximate tolerance limits for $\hat{C}_p$ based on Range calculated as

Approximate upper tolerance limit**:** $\left\{\frac{J_2\,\hat{C}^{\alpha}_{pamR} + J_2\,\hat{C}^{\alpha}_{pdmR}}{2}\right\}$

Center line: $\left\{C^{\alpha}_{pmR} = \frac{\hat{C}^{\alpha}_{pa} + \hat{C}^{\alpha}_{pd}}{2}\right\}$ (18)

Approximate lower tolerance limit: $\left\{\frac{J_1\,\hat{C}^{\alpha}_{pamR} + J_1\,\hat{C}^{\alpha}_{pdmR}}{2}\right\}$

The condition of the process control can be defined by

$$\text{process control} = \begin{cases} \text{incontrol} & J_1 C^{\alpha}_{pmR} \le s^{\alpha}_{mRj} \le J_2 C^{\alpha}_{pmR} \\ \text{out} - \text{of control} & \text{otherwise} \end{cases} \tag{19}$$

**Applications of α- Level Fuzzy Midrange Approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

An application was given by Sentruk and Erginel [9] using trapezoidal fuzzy numbers on controlling piston inner diameters in compressors. The same data have been considered with the first fifteen samples each with size 5 (the total measurements is 5 X 15 = 75) The upper and lower specification limits of the process are defined as approximately 5.7 and approximately 5.1, respectively.

These measurements are converted into trapezoidal fuzzy numbers and given in Table 1. Fuzzy control limits are calculated according to the procedures given in the previous sections.

**Table 1: The Trapezoidal Fuzzy Measurement Values & the Fuzzy Ranges**

| No. | $X_a$ | | | | | $X_b$ | | | | | $X_c$ | | | | | $X_d$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| 1 | 5.71 | 5.5 | 5.43 | 5.2 | 5.51 | 5.73 | 5.57 | 5.45 | 5.25 | 5.53 | 5.75 | 5.6 | 5.46 | 5.27 | 5.55 | 5.76 | 5.62 | 5.47 | 5.28 | 5.56 |
| 2 | 5.41 | 5.52 | 5.25 | 5.51 | 5.65 | 5.43 | 5.57 | 5.29 | 5.53 | 5.69 | 5.44 | 5.58 | 5.3 | 5.56 | 5.7 | 5.45 | 5.59 | 5.31 | 5.58 | 5.71 |
| 3 | 5.25 | 5.51 | 5 | 5.2 | 5.31 | 5.29 | 5.53 | 5.13 | 5.25 | 5.33 | 5.32 | 5.54 | 5.17 | 5.28 | 5.37 | 5.33 | 5.55 | 5.19 | 5.29 | 5.39 |
| 4 | 5.42 | 5.26 | 5.42 | 5.49 | 5.6 | 5.51 | 5.31 | 5.44 | 5.55 | 5.64 | 5.62 | 5.38 | 5.46 | 5.58 | 5.71 | 5.75 | 5.4 | 5.51 | 5.65 | 5.76 |
| 5 | 5.19 | 5.18 | 5.25 | 5.21 | 5.52 | 5.3 | 5.2 | 5.28 | 5.27 | 5.57 | 5.32 | 5.31 | 5.32 | 5.3 | 5.6 | 5.41 | 5.35 | 5.4 | 5.39 | 5.65 |
| 6 | 5.36 | 5.31 | 5.18 | 5.38 | 5.26 | 5.42 | 5.4 | 5.23 | 5.48 | 5.33 | 5.63 | 5.55 | 5.31 | 5.47 | 5.42 | 5.71 | 5.62 | 5.32 | 5.68 | 5.52 |
| 7 | 5.26 | 5.53 | 5.41 | 5.28 | 5.19 | 5.27 | 5.57 | 5.46 | 5.29 | 5.26 | 5.31 | 5.62 | 5.49 | 5.32 | 5.3 | 5.46 | 5.59 | 5.52 | 5.38 | 5.39 |
| 8 | 5.43 | 5.28 | 5.44 | 5.5 | 5.58 | 5.52 | 5.33 | 5.47 | 5.56 | 5.62 | 5.63 | 5.4 | 5.5 | 5.6 | 5.69 | 5.76 | 5.42 | 5.55 | 5.56 | 5.74 |
| 9 | 5.69 | 5.45 | 5.32 | 5.19 | 5.45 | 5.72 | 5.49 | 5.35 | 5.22 | 5.48 | 5.75 | 5.56 | 5.41 | 5.28 | 5.51 | 5.76 | 5.61 | 5.45 | 5.31 | 5.56 |
| 10 | 5.31 | 5.26 | 5.14 | 5.4 | 5.31 | 5.35 | 5.29 | 5.21 | 5.43 | 5.36 | 5.42 | 5.32 | 5.26 | 5.46 | 5.38 | 5.46 | 5.39 | 5.32 | 5.52 | 5.42 |
| 11 | 5.28 | 5.5 | 5.41 | 5.31 | 5.35 | 5.32 | 5.56 | 5.46 | 5.34 | 5.41 | 5.36 | 5.62 | 5.48 | 5.38 | 5.44 | 5.43 | 5.65 | 5.52 | 5.44 | 5.49 |
| 12 | 5.43 | 5.22 | 5.15 | 5.34 | 5.48 | 5.46 | 5.26 | 5.18 | 5.36 | 5.52 | 5.48 | 5.28 | 5.21 | 5.41 | 5.56 | 5.53 | 5.31 | 5.26 | 5.44 | 5.62 |
| 13 | 5.46 | 5.35 | 5.35 | 5.22 | 5.28 | 5.52 | 5.39 | 5.42 | 5.28 | 5.32 | 5.56 | 5.43 | 5.48 | 5.34 | 5.36 | 5.62 | 5.48 | 5.54 | 5.38 | 5.42 |
| 14 | 5.41 | 5.36 | 5.52 | 5.51 | 5.38 | 5.44 | 5.4 | 5.56 | 5.54 | 5.42 | 5.48 | 5.43 | 5.62 | 5.62 | 5.46 | 5.52 | 5.48 | 5.68 | 5.71 | 5.49 |
| 15 | 5.62 | 5.48 | 5.42 | 5.18 | 5.41 | 5.64 | 5.55 | 5.46 | 5.23 | 5.46 | 5.68 | 5.59 | 5.49 | 5.26 | 5.49 | 5.73 | 5.63 | 5.54 | 5.31 | 5.53 |

**α- Level Fuzzy Midrange Approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

α- level Fuzzy midrange approximate tolerance limits for $\hat{C}_p$ based on Range calculated as

Approximate upper tolerance limit: $\left\{\frac{J_2\,\hat{C}^{\alpha}_{pamR} + J_2\,\hat{C}^{\alpha}_{pdmR}}{2}\right\} = \{1.26693\}$

Center line: $\left\{C^{\alpha}_{pmR} = \frac{\hat{C}^{\alpha}_{pa} + \hat{C}^{\alpha}_{pc}}{2}\right\} = \{0.70307\}$

Approximate lower tolerance limit: $\left\{\frac{J_1\,\hat{C}^{\alpha}_{pa}mR + J_1\,\hat{C}^{\alpha}_{pdmR}}{2}\right\} = \{0.58776\}$

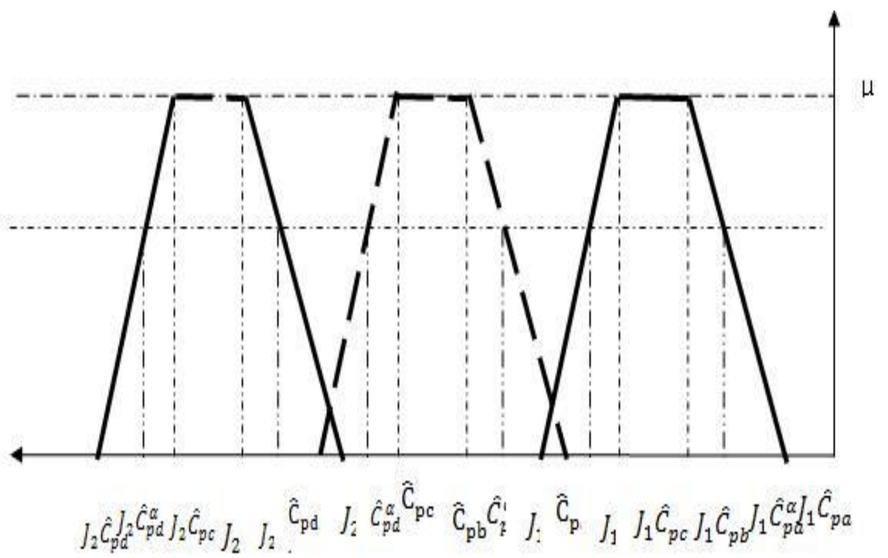


**Figure 2: α- Level Fuzzy Midrange Approximate Tolerance Limits for $\ddot{C}_p$ Based on Range**

**Table 2: α- Level Fuzzy Midrange Approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

| Sample No. | $\hat{C}_{pa}$ | $\hat{C}_{pb}$ | $\hat{C}_{pc}$ | $\hat{C}_{pd}$ | $s^{\alpha}_{mRj}$ | $0.58776 \le s^{\alpha}_{mRj} \le 1.26693$ |
|---|---|---|---|---|---|---|
| 1 | 0.45608 | 0.48458 | 0.48458 | 0.47959 | 0.47959 | Out of control |
| 2 | 0.5815 | 0.5815 | 0.5815 | 0.5815 | 0.58150 | Out of control |
| 3 | 0.45608 | 0.5815 | 0.62865 | 0.5678 | 0.58618 | Out of control |
| 4 | 0.68412 | 0.70485 | 0.70485 | 0.70122 | 0.69094 | In control |
| 5 | 0.68412 | 0.62865 | 0.77533 | 0.66403 | 0.71170 | In control |
| 6 | 1.1630 | 0.9304 | 0.72687 | 0.93549 | 0.84651 | In control |
| 7 | 0.68412 | 0.75032 | 0.72687 | 0.73463 | 0.79364 | In control |
| 8 | 0.77533 | 0.80207 | 0.80207 | 0.79739 | 0.77675 | In control |
| 9 | 0.4652 | 0.4652 | 0.49489 | 0.4704 | 0.48390 | Out of control |
| 10 | 0.89462 | 1.05727 | 1.1630 | 1.04731 | 1.08167 | In control |
| 11 | 1.05727 | 0.96917 | 0.89462 | 0.97154 | 0.97577 | In control |
| 12 | 0.70485 | 0.68412 | 0.66457 | 0.68432 | 0.67474 | In control |
| 13 | 0.96917 | 0.96917 | 1.05727 | 0.98459 | 0.99780 | In control |
| 14 | 1.45375 | 1.45375 | 1.22421 | 1.41358 | 1.30172 | Out of control |
| 15 | 0.52864 | 0.56732 | 0.55381 | 0.55818 | 0.55379 | Out of control |

$s^{\alpha}_{mRj}$ have been calculated for all the 15 samples with respect to different operators by using equations (13) and are given in the table 2. As shown in the table 2, traditional mean and range chart of Shewhart show that the process is in control but the process capability index decreased and increased in samples like 1, 2,3, 9 and 14 which indicating the process is out of control and need of investigations. The - level Fuzzy midrange approximate tolerance limits for $\hat{C}_p$ based on Range can be used to control the process.

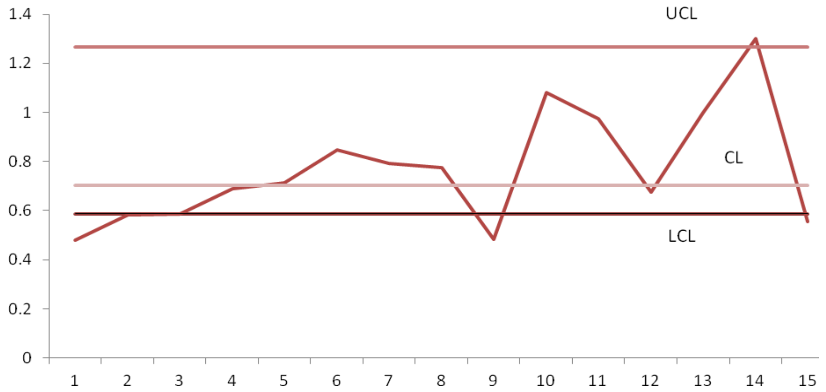


**Figure 1: α- Level Fuzzy Midrange Approximate Tolerance Limits for $\hat{C}_p$ Based on Range**

## CONCLUSIONS

This paper shows that the α - level Fuzzy midrange approximate tolerance limits for $\hat{C}_p$ based on Range can be used to control the process. The fuzzy set theory is suitable for process capability indices as a new different approach to control the process. This study shows that the Capability indices based on the range combines the customer requirement and process performance. The study of fuzzy process capability indices provide evidence of improvement compared with the traditional charts and show more flexibility in tolerance control limits. This study can be extended by other fuzzy transformation techniques.

## REFERENCES


1. Bahram Sadeghpour Gildeh, (2011) " Measurment error effects on the performance of process capability index based on fuzzy tolerance interval," Annals of Fuzzy Mathematics and informatics, Vol. 2, No. 1, pp. 17 – 32.
2. Bernardo J.M., Irony T. Z., (1996), "A general Multivariate Bayesian Process capability index," Journal of Royal Statistical Society, Vol. 45, pp. 487 – 502.
3. Carr, W.E.,(1991), "A new process capability index: Parts per Million," Quality progress, Vol. 24, pp. 152.
4. Chen K.S., Chen K.L., Li. R.K. (2005), "Contract manufacturer selection by using the process incapability index Cpp," International Journal of Advanced manufacturing Technology" Vol. 26, pp. 686 – 692.
5. Cengiz Kahraman., Ihsan Kaya., (2011), "Fuzzy estimates of process incapability index," Proceedings of the world Congress on Engineering, Vol. II, London, U.k.
6. Greenwich M., Jahr-Schaffrath B.L.,(1995), "A process incapability index," International journal of quality and reliability Management, Vol. 12(4), pp. 58 – 71.
7. Hsin-Lin Kuo, (2008), "Approximate confidence Bounds for the process capability index Cpm based on subsamples," International Journal of Information and Management Sciences, Vol. 19(4), pp. 601 – 620.
8. Hsin-Lin Kuo, (2010), "Approximate Tolerance limits for $C_p$ Capability chart based on Range", ProbStat Forum, Volume 03, pp 145 – 157.
9. Kane V.E., (1986), "Process capability indices", journal of quality technology, vol. 18(1), pp. 41-52.
10. Montgomery, D.C. (2001), Introduction to statistical quality control (4th edition), New York.
11. Patnaik, P.B. (1950), "The use of mean range as an estimator of variance is statistical test,", Biometriika, Vol. 37, pp. 78 – 92.
12. Selva Arul Pandian.S., and Puthiyanayagam.P (2012), "The approximate tolerance limits for $\hat{C}_p$ capability chart based on range using α -cuts triangular fuzzy numbers" 'international journal of advanced scientific and technical research', Issue 2 volume 5, pp 377-386
13. Sevil Senturk and Nihal Erginel (2009), "Development of fuzzy $\tilde{\bar{X}} - \tilde{R}$ and $\tilde{\bar{X}} - \tilde{S}$ Control charts using α -Cuts". Information Sciences, 179(10), pp. 1542-1551.
14. Sullivan L.P., (1984), Reducing variability: a new approach to quality, quality progress Vol. 17 pp. 15 – 21.
15. Vannman K, (1995), "A unified approach to capability indices," Statistical Sciences, Vol. 5, pp. 805 – 820.
16. Vannman, K. (2002), Discussion, Journal of Quality Technology, 34(1) pp. 40 – 42.